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Bouncing and accelerating solutions in nonlocal stringy models

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Abstract: A general class of cosmological models driven by a nonlocal scalar field inspired by string field theories is studied. In particular cases the scalar field is a string dilaton or a string tachyon. A distinguished feature of these models is a crossing of the phantom divide. We reveal the nature of this phenomena showing that it is caused by an equivalence of the initial nonlocal model to a model with an infinite number of local fields some of which are ghosts. Deformations of the model that admit exact solutions are constructed. These deformations contain locking potentials that stabilize solutions. Bouncing and accelerating solutions are presented.

Keywords: Cosmology of Theories beyond the SM, String Field Theory.

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Contents

1. Introduction

Field theories which violate the null energy condition (NEC) [1, 2] are of interest for the solution of the cosmological singularity problem and for the construction of cosmological dark energy models with the state parameter $w < -1$.

One of the first attempts to apply string theory to cosmology [3] was related to the problem of the cosmological singularity [2]. A possible way to avoid cosmological singularities consists of dealing with nonsingular bouncing cosmological solutions. In these scenarios the Universe contracts before the bounce [4]. Such models have strong coupling and higher-order string corrections are inevitable. It is important to construct nonsingular bouncing cosmological solutions in order to make a concrete prediction of bouncing cosmology.

Present cosmological observations [5] do not exclude an evolving dark energy (DE) state parameter w, whose current value is less than -1, that means the violation of the NEC (see [6, 7] for a review of DE problems and [8] for a search for a super-acceleration phase of the Universe).

A simple possibility to violate NEC is just to deal with a phantom field. The phantom field is unstable. There are general arguments that coupled scalar-gravity models violating the NEC are unstable ([9]–[13] and refs. therein). At the same time a phantom model could be an approximation to a nonlocal model that has no problems with instability [14]. A simple example of such a model is a model with a scalar nonlocal action $(e^{-\Box_g}\phi)^2$. In the second order derivative approximation $e^{-\Box_g} \approx 1-\Box_g$ this model is equivalent to a phantom one but does not have problems with instability. This type of models does appear in String Field Theory (SFT)(see [15] for a review) and in the p-adic string models [16]. The model with the particular kinetic term mentioned above is in fact realized in the p-adic string near a perturbative vacuum and is expected to be realized in the Vacuum String Field Theory (VSFT) [17].

The purpose of this paper is the study of this type of models. As a model we consider a SFT inspired nonlocal dilaton action. Distinguished features of the model are the invariance of the action under the shift of the dilaton field to a constant as well as a presence of infinite number of higher derivatives terms. A more general family of nonlocal models loosing the invariance of the nonlocal dilaton is also considered. For special values of the parameters the models describe linear approximations to the cubic bosonic or nonBPS fermionic SFT nonlocal tachyon models, or p-adic string models [14, 18]–[24]. The NonBPS fermionic string field tachyon nonlocal model has been considered as a candidate for the dark energy [14]. Several string-inspired and braneworld dark energy models have been recently proposed (see for example [25]–[27] and refs. therein). About a study of the tachyon dynamics with the Born-Infeld action see [28 – 30].

We discuss a possibility to stabilize the model that violates the NEC in the flat space-time at the cost of adding extra interaction terms in the Friedmann background. One of the lessons from a study nonlocal dynamics in the flat case is a sensitivity of the stability problem to the form of the interaction term [31 – 35, 18, 36]. We use the Weierstrass product to present the nonlocal field in terms of an infinite number of local fields [22]. Some of these local fields are ghosts, which violate the NEC and are unstable. The model is linear and admits exact solutions in the flat space-time. In non-flat case we get the same exact solutions after a deformation of the model. We used a similar approach to construct effective SFT inspired phantom models [37, 18, 38].

Another recently proposed model which violates the NEC and has higher derivatives is the ghost-condensation model [39]. Vector-scalar and tensor-scalar models that violate NEC and are stable in some region have been proposed in [40, 41], respectively.

The paper is organized as follows. In section 2 we describe our strategy to the study of stringy inspired models. In section 3 we present general solutions of the models in the flat case. Then we use some approximation to study these dynamics in the Friedmann metric and discuss cosmological properties of the constructed solutions.

2. Set up

In this paper we consider a model of gravity coupling with a nonlocal scalar field which induced by strings field theory

$$S=\int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R+\frac{M_s^4}{g_4}\left(\frac{1}{2}\phi\, F(-\Box_g/M_s^2)\phi-\Lambda'\right)\right), \qquad (2.1)$$

where g is the metric, $\Box_g=\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$, M_p is a mass Planck, M_s is a characteristic string scale related with the string tension α', $M_s=1/\sqrt{\alpha'}$, ϕ is a dimensionless scalar field (tachyon or dilaton), g_4 is a dimensionless four dimensional effective coupling constant related with the ten dimensional string coupling constant g_0 and the compactification scale. $\Lambda=\frac{M_s^4}{g_4}\Lambda'$ is an effective four dimensional cosmological constant.

The form of the function F is inspired by a nonlocal action appeared in string field theories. In particular cases

$$F(z)=-\xi^2z+1-ce^{-2z}, \qquad (2.2)$$

ξ is a real parameter and c is a positive constant. Using dimensional space-time variables and after a rescaling we can rewrite (2.1) for F given by (2.2) as follows

$$\int d^4x\sqrt{-g}\left(\frac{m_p^2}{2}R+\frac{\xi^2}{2}\phi\Box_g\phi+\frac{1}{2}(\phi^2-c\Phi^2)-\Lambda'\right), \qquad (2.3)$$

where $\Phi=e^{\Box_g}\phi$ and $m_p^2=g_4M_p^2/M_s^2$. Generally speaking the string scale does not coincide with the Plank mass [21, 23]. This gives a possibility to get a realistic value of Λ [21].

The form of the term $(e^{\Box_g}\phi)^2$ is analogous to the form of the interaction in the action for the string field tachyon in non-flat background [14], which is a generalization of the SFT tachyon interaction term in a flat background [42, 43, 15, 32, 33]. At some particular values of ξ^2 and c this action appears in a linear approximation to SFT actions [44]–[48] and in a non-flat background has been considered in [14, 18–20, 24]. The case of the open Cubic Superstring Field Theory (CSSFT) [14, 20] tachyon corresponds to $\xi^2=-1/\left(4\ln\left(\frac{4}{3\sqrt{3}}\right)\right)\approx 0.9556$ and $c=3$. We consider in detail action (2.3) at $c=1$, which is invariant under translation $\phi\to\phi+const$.

We take the metric in the form

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right) \tag{2.4}$$

and get the following equation of motion for the space homogeneous scalar field ϕ:

$$F(-\mathcal{D})\phi = 0, \tag{2.5}$$

where

$$\mathcal{D} \equiv -\partial_t^2 - 3H(t)\partial_t, \quad H = \frac{\dot{a}}{a} \quad \text{and} \quad \dot{a} \equiv \partial_t a. \tag{2.6}$$

The Friedmann equations have the following form

$$\begin{aligned} 3H^2 &= \frac{1}{m_p^2}\,\mathcal{E}, \\ 3H^2 + 2\dot{H} &= -\frac{1}{m_p^2}\,\mathcal{P}, \end{aligned} \tag{2.7}$$

where the energy and the pressure are obtained from the action (2.1) using standard formula

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}, \qquad T_{\mu\nu} = \mathrm{diag}\{\mathcal{E}, \mathcal{P}, \mathcal{P}, \mathcal{P}\}. \tag{2.8}$$

For the case of F given by (2.2) the energy and the pressure have additional nonlocal terms $\mathcal{E}_{nl1}$ and $\mathcal{E}_{nl2}$ [33, 49, 50]

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_k + \mathcal{E}_p + \mathcal{E}_{nl2} + \mathcal{E}_{nl1} + \Lambda', \\ \mathcal{P} &= \mathcal{E}_k - \mathcal{E}_p + \mathcal{E}_{nl2} - \mathcal{E}_{nl1} - \Lambda'. \end{aligned} \tag{2.9}$$

Nonlocal term $\mathcal{E}_{nl1}$ plays a role of an extra potential term and $\mathcal{E}_{nl12}$ a role of an extra kinetic term. The explicit form of the terms in the r.h.s. of (2.9) is

$$\begin{aligned} \mathcal{E}_k &= \frac{\xi^2}{2}(\partial\phi)^2, \qquad \mathcal{E}_p = -\frac{1}{2}\left(\phi^2 - c(e^{\mathcal{D}}\phi)^2\right), \\ \mathcal{E}_{nl1} &= c\int_0^1 \left(e^{(1+\rho)\mathcal{D}}\phi\right)\left(-\mathcal{D}e^{(1-\rho)\mathcal{D}}\phi\right)d\rho, \\ \mathcal{E}_{nl2} &= -c\int_0^1 \left(\partial e^{(1+\rho)\mathcal{D}}\phi\right)\left(\partial e^{(1-\rho)\mathcal{D}}\phi\right)d\rho. \end{aligned} \tag{2.10}$$

Our strategy is the following:
First, we study the dynamics of the model (2.3) in the flat case.

- We show that solutions have the form of plane waves. There are special values of parameters for which plane waves should be multiplied on linear functions. We calculate the energy and pressure on the corresponding solutions.
- We present dynamics of the nonlocal model in terms of an infinite number of local fields [22] and show that the energy and pressure densities of the nonlocal model are reproduced by the energy and pressure densities of the corresponding local models.

For this purpose we use the Weierstrass product representation for the function F in (2.1),

$$F(z) = e^{f(z)} \prod_n \left(1 - \frac{z}{\alpha_n^2}\right), \tag{2.11}$$

where α_n^2 are complex numbers, and represent the flat analog of (2.1) as

$$S_{\text{flat}} = \frac{1}{2}\int d^4x \phi F(-\Box)\phi \sim \frac{1}{2}\sum \left[\epsilon_n \psi_n e^{f(-\Box)}(\Box + \alpha_n^2)\psi_n + c.c.\right], \tag{2.12}$$

where $\Box$ is the d'Alembertian in the flat space-time.

- We consider approximated models obtained by a truncation of number of local fields.

Then we consider the Friedmann Universe. There are two ways to study dynamics in the Friedmann metric:

- One can use the found expressions for the energy and pressure in the flat case, $E(\phi_0)$ and $P(\phi_0)$, to calculate the corresponding Hubble parameter H_0, then using this Hubble parameter calculate a perturbation of the flat solution of equation of motion and so on:

$$\phi = \phi_0 + \phi_1(H_0) + \dots, \qquad H = H_0(\phi_0) + H_1(\phi_0, \phi_1) + \dots \tag{2.13}$$

- One can search for deformations of the model that admit the same exact solutions as in the flat case and try to argue that the deformed models describe the initial model with a good accuracy.

Both ways permit to find the first approximations to the models (2.5), (2.7). The first way have been used in [20]. In this paper we will follow the second way.

To this goal we use a representation of nonlocal dynamics given by action (2.1) in terms of local fields

$$\int d^4x\sqrt{-g}\left(\frac{m_p^2}{2}R + \frac{1}{2}\sum \left[\epsilon_n \psi_n e^{f(-\Box_g)}(\Box_g + \alpha_n^2)\psi_n + \Lambda' + c.c.\right]\right). \tag{2.14}$$

We perform a deformation of this model by several steps. First, we consider an approximation to the model (2.14) in the form

$$\int d^4x\sqrt{-g}\left(\frac{m_p^2}{2}R + \sum \left[\frac{\epsilon_n}{2}\psi_n e^{f(\alpha_n^2)}(\Box_g + \alpha_n^2)\psi_n + \Lambda' + c.c.\right]\right) \tag{2.15}$$

Second, we restrict a number of local fields and, third, we add potentials of the order $1/m_p^2$ in which Λ' is also included:

$$\int d^4x\sqrt{-g}\left(\frac{m_p^2}{2}R + \sum \left[\frac{\epsilon_n}{2}\psi_n e^{f(\alpha_n^2)}(\Box_g + \alpha_n^2)\psi_n + c.c.\right] - \mathcal{V}(\psi_1, \dots, \psi_n)\right). \tag{2.16}$$

such that solutions of the field equations in the non-flat case are the same as the flat case.

Finally, we find the corresponding scale factor $a(t)$ and study cosmological properties of approximated solutions to our model.

3. Flat dynamics

3.1 General solutions

3.1.1 Roots of the characteristic equation

In the flat case the action (2.1) has the following form:

$$S_{\text{flat}} = \frac{1}{2}\int d^4x\phi F(-\Box)\phi. \tag{3.1}$$

Equation of motion on the space-homogeneous configurations (2.5) is reduced to the following linear equation:

$$F(\partial^2)\phi = 0. \tag{3.2}$$

A plane wave

$$\phi = e^{\alpha t} \tag{3.3}$$

is a solution of (3.2) if α is a root of the characteristic equation

$$F(\alpha^2) = 0. \tag{3.4}$$

For a case of F given by (2.2) equation (3.2) has the following form

$$-\xi^2\partial^2\phi + \phi - ce^{-2\partial^2}\phi = 0. \tag{3.5}$$

This equation has an infinite number of derivatives and can be treated as a pseudodifferential as well as an integral equation [35].

The corresponding characteristic equation:

$$F(\alpha^2) \equiv -\xi^2\alpha^2 + 1 - ce^{-2\alpha^2} = 0 \tag{3.6}$$

has the following solutions

$$\alpha_n = \pm\frac{1}{2\xi}\sqrt{4 + 2\xi^2 W_n\left(-\frac{2ce^{-2/\xi^2}}{\xi^2}\right)}, \quad n = 0, \pm 1, \pm 2, \ldots \tag{3.7}$$

where W_n is the n-s branch of the Lambert function satisfying a relation $W(z)e^{W(z)} = z$. The Lambert function is a multivalued function, so eq. (3.6) has an infinite number of roots.

Parameters ξ and c are real, therefore if α_n is a root of (3.6), then the adjoined number α_n^* is a root as well. Note that if α_n is a root of (3.6), then $-\alpha_n$ is a root too. In other words, equation (3.6) has quadruples of complex roots

$$\alpha_{n,\pm\pm} = \pm\mathrm{Re}(\alpha_n) \pm i\mathrm{Im}(\alpha_n). \tag{3.8}$$

If $\alpha^2 = \alpha_0^2$ is a multiple root, then at this point $F(\alpha_0^2) = 0$ and $F'(\alpha_0^2) = 0$. These equations give that

$$\alpha_0^2 = \frac{1}{\xi^2} - \frac{1}{2}, \tag{3.9}$$

hence α_0^2 is a real number and all multiple roots of $F(\alpha_0^2) = 0$ are either real or pure imaginary. The multiple roots exist if and only if

$$c = \frac{\xi^2}{2e} e^{2/\xi^2}. \tag{3.10}$$

Real roots for any ξ and c, except $\xi^2 = 0$ and $c = \infty$, are no more then double degenerated, because $F''(\alpha_0^2) \neq 0$.

Summing up we note that according as the values of parameters c and ξ^2 there exist the following types of the general real solution of (3.5):

- If $c \neq \frac{\xi^2}{2e} e^{2/\xi^2}$ and $c \neq 1$ then the general real solution is

$$\phi = \sum_n R_n e^{m_n t} + \sum_n \left(C_n e^{\alpha_n t} + C_n^* e^{\alpha_n^* t} \right), \tag{3.11}$$

  where R_n and C_n are arbitrary real and complex numbers respectively.

- If $c = \frac{\xi^2}{2e} e^{2/\xi^2} > 1$, then to get the general real solution one has to add to (3.11)

$$\phi_0 = \tilde{R}_1 t e^{m_0 t} + \tilde{R}_2 t e^{-m_0 t}, \qquad m_0 = \sqrt{\frac{1}{\xi^2} - \frac{1}{2}} \quad \text{if} \quad \xi^2 < 2, \tag{3.12}$$

$$\phi_0 = \tilde{C}_1 t e^{i\alpha_0 t} + \tilde{C}_1^* t e^{-i\alpha_0 t}, \qquad \alpha_0 = i\sqrt{\frac{1}{2} - \frac{1}{\xi^2}} \quad \text{if} \quad \xi^2 > 2. \tag{3.13}$$

- If $c = 1$ then to get the general real solution one has to add to (3.11)

$$\phi_0 = C_1 t + C_0, \qquad \text{if} \quad \xi^2 \neq 2, \tag{3.14}$$

$$\phi_0 = C_3 t^3 + C_2 t^2 + C_1 t + C_0, \qquad \text{if} \quad \xi^2 = 2. \tag{3.15}$$

3.1.2 Real roots of the characteristic equation

For some values of the parameters ξ and c eq. (3.6) has real roots. To mark out real values of α we will denote real α as m: $m = \alpha$.

To determine values of the parameters at which eq. (3.6) has real roots we rewrite this equation in the following form:

$$\xi^2 = g(m^2, c), \quad \text{where} \quad g(m^2, c) = \frac{e^{2m^2} - c}{m^2 e^{2m^2}}. \tag{3.16}$$

The dependence of $g(m, c)$ on m for different c is presented in figure (1). This function has a maximum at $m_{\max}^2$

$$m_{\max}^2 = -\frac{1}{2} - \frac{1}{2} W_{-1}\left(-\frac{e^{-1}}{c} \right), \tag{3.17}$$

provided c is such that $W_{-1}\left(-\frac{e^{-1}}{c}\right) < -1$, in the other words $0 < c < 1$.

There are three different cases (see figure 1).

- If $c < 1$, then eq. (3.6) has two simple real roots: $m = \pm m_1$ for any values ξ.

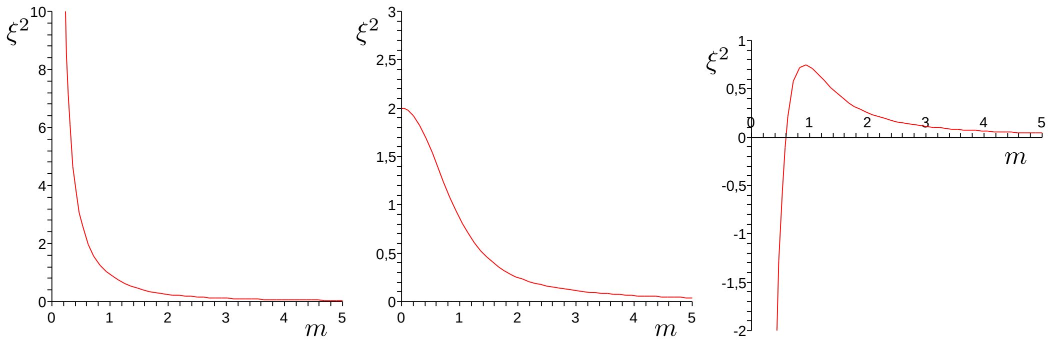

Figure 1: The dependence of function $g(m^2, c)$, which is equal to ξ^2, on m at $c = 1/2$ (left), $c = 1$ (center) and $c = 2$ (right).

- If $c = 1$, then eq. (3.6) has a zero root. Nonzero real roots exist if and only if $\xi^2 > 2$.
- If $c > 1$, then eq. (3.6) has
  - no real roots for $\xi^2 > \xi^2_{\max}$, where

$$\xi^2_{\max} = \frac{1 - ce^{-2m^2_{\max}}}{m^2_{\max}} = -\frac{2}{W_{-1}(-e^{-1}/c)} \tag{3.18}$$

  - two real double roots $m = \pm m_{\max}$ for $\xi^2 = \xi^2_{\max}$
  - four real single roots for $\xi^2 < \xi^2_{\max}$. In this case we have the following restriction on real roots: $m^2 > \frac{1}{2}\ln c$.

3.1.3 Pure imaginary roots of the characteristic equation

For some values of the parameters ξ and c eq. (3.6) has a pair of pure imaginary roots. Let us introduce a new real variable $\mu = i\alpha$. From (3.5) we obtain

$$(\xi^2\mu^2 + 1)e^{-2\mu^2} = c. \tag{3.19}$$

This equation is equivalent to

$$\xi^2 = \tilde{g}(\mu^2, c), \quad \text{where} \quad \tilde{g}(\mu^2, c) = \frac{c - e^{-2\mu^2}}{\mu^2 e^{-2\mu^2}}. \tag{3.20}$$

The dependence of $\tilde{g}(\mu^2, c)$ on μ for different c is presented in figure 2.

For different c we have:

- For $c < 1$ there are two real simple roots $\mu = \pm\mu_1$,
- For $c = 1$ nonzero real roots exist only if $\xi^2 > 2$,
- For $c > 1$ real roots exist if and only if

$$\xi \geqslant \xi_{\min} = -\frac{2}{W_0\left(-\frac{e^{-1}}{c}\right)}. \tag{3.21}$$

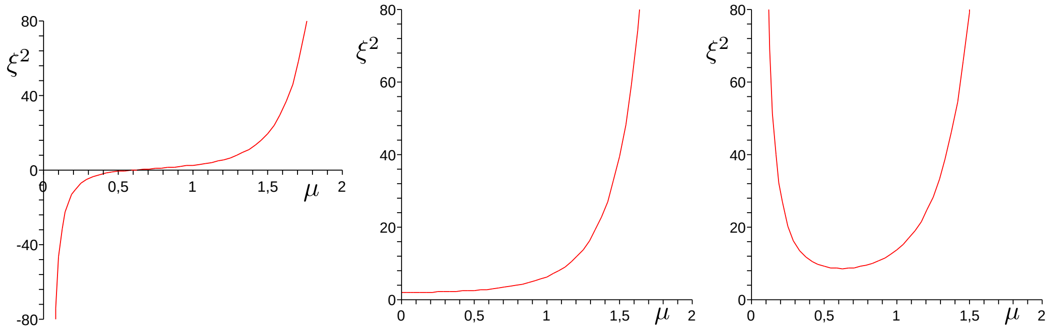

Figure 2: The dependence of function $\tilde{g}(\mu^2, c)$, which is equal to ξ^2, on μ at $c = 1/2$ (right), $c = 1$ (center) and $c = 2$ (left).

If $\xi = \xi_{\min}$, then there exist two double real roots: $\mu = \pm\mu_{\min}$, where

$$\mu_{\min} = \frac{1}{2}\sqrt{2 + 2W_0\left(-\frac{e^{-1}}{c}\right)}. \tag{3.22}$$

At $\xi > \xi_{\min}$ eq. (3.19) has four real simple roots.

3.1.4 Roots at the SFT inspired values on ξ^2 and c

Let us consider a special values of ξ^2 and c, which have been obtain in the SFT inspired cosmological model [14, 20]. From the action for the tachyon in the CSSFT [45] the following equation has been obtained [20]:

$$(-\xi_0^2\tilde{\alpha}^2 + 1) = 3e^{-\tilde{\alpha}^2/4}, \tag{3.23}$$

where

$$\xi_0^2 = -\frac{1}{4\ln\left(\frac{4}{3\sqrt{3}}\right)} \approx 0.9556. \tag{3.24}$$

Substituting $\tilde{\alpha} = 2\sqrt{2}\alpha$, we obtain eq. (3.6) with $\xi^2 = 8\xi_0^2$ and $c = 3$. From (3.10) it is follows that all roots are simple. We obtain that $\xi^2_{\min} > \xi^2 > \xi^2_{\max}$, so there exist neither real roots nor pure imaginary roots.

3.2 Energy density and pressure

3.2.1 General formula

Equation (3.5) has the conserved energy (compare with [49, 50, 18]), which is defined by the formula that is a flat analog of (2.9). The energy density is as follows:

$$E = E_k + E_p + E_{nl1} + E_{nl2}, \tag{3.25}$$

where

$$E_k = \frac{\xi^2}{2}(\partial\phi)^2, \qquad E_p = -\frac{1}{2}\phi^2 + \frac{c}{2}\Phi^2, \tag{3.26}$$
$$E_{nl1} = c\int_0^1 \left(e^{-\rho\partial^2}\Phi\right)\left(\partial^2(e^{\rho\partial^2}\Phi)\right)d\rho, \qquad E_{nl2} = -c\int_0^1 \left(\partial(e^{-\rho\partial^2}\Phi)\right)\left(\partial(e^{\rho\partial^2}\Phi)\right)d\rho.$$

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For the pressure

$$P = E_k - E_p - E_{nl1} + E_{nl2}, \tag{3.27}$$

we have the following explicit form

$$P = \frac{\xi^2}{2}(\partial\phi)^2 + \frac{1}{2}\phi^2 - \frac{c}{2}\Phi^2 - c\int_0^1 \left\{\left(e^{-\rho\partial^2}\Phi\right)\left(\partial^2(e^{\rho\partial^2}\Phi)\right) - \left(\partial(e^{-\rho\partial^2}\Phi)\right)\left(\partial(e^{\rho\partial^2}\Phi)\right)\right\} d\rho. \tag{3.28}$$

Let us calculate the energy density and pressure for the following solution

$$\phi = \sum_{n=1}^{N} C_n e^{\alpha_n t}, \tag{3.29}$$

where N is a natural number, C_n are some constant and α_n are solutions to eq. (3.6).

For $N = 1$ and

$$\phi = Ce^{\alpha t} \tag{3.30}$$

we obtain

$$E(Ce^{\alpha t}) = 0, \tag{3.31}$$

$$P(Ce^{\alpha t}) = C^2 p_\alpha e^{2\alpha t}. \tag{3.32}$$

Hereafter we denote the energy density and pressure of function $\phi(t)$ as the functionals $E(\phi)$ and $P(\phi)$, respectively, and use the following notation

$$p_\alpha \equiv \alpha^2\left(\xi^2 - 2 + 2\xi^2\alpha^2\right). \tag{3.33}$$

For $N = 2$ and

$$\phi = C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}, \tag{3.34}$$

where α_1 and α_2 are different roots of (3.6) we have (see appendix A for details)

$$E(C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}) = -2C_1 C_2 p_{\alpha_1}, \qquad \text{at} \quad \alpha_2 = -\alpha_1, \tag{3.35}$$

and

$$E(C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}) = 0, \qquad \text{at} \quad \alpha_2 \neq -\alpha_1. \tag{3.36}$$

The pressure $P(\phi)$ for solution (3.34) is

$$P(C_1 e^{-\alpha t} + C_2 e^{\alpha t}) = \left(C_1^2 e^{-2\alpha t} + C_2^2 e^{2\alpha t}\right) p_\alpha. \tag{3.37}$$

In the general case we have

$$E\left(\sum_{n=1}^{N} C_n e^{\alpha_n t}\right) = -2\sum_{n=1}^{N}\sum_{k=n+1}^{N} C_n C_k p_{\alpha_n}\delta_{\alpha_n,-\alpha_k}, \tag{3.38}$$

where

$$\delta_{\alpha_n,-\alpha_k} = \begin{cases} 1, & \alpha_n = -\alpha_k, \\ 0, & \alpha_n \neq -\alpha_k. \end{cases} \tag{3.39}$$

Note that all summands in (3.38) are integrals of motion, therefore, we explicitly show that $E\left(\sum_{n=1}^{N} C_n e^{\alpha_n t}\right)$ is an integral of motion. From formula (3.38) we see that the energy density is a sum of the crossing terms. At the same time the pressure is a sum of "individual" pressures and has no crossing term. In the case of an arbitrary finite number of summands the pressure is as follows:

$$P\left(\sum_{n=1}^{N} C_n e^{\alpha_n t}\right) = \sum_{n=1}^{N} C_n^2 P\left(e^{\alpha_n t}\right) = \sum_{n=1}^{N} C_n^2 p_{\alpha_n} e^{2\alpha_n t}. \tag{3.40}$$

If the parameters ξ^2 and c are such that the characteristic equation (3.6) have double roots, then eq. (3.5) has the following solution

$$\phi_0(t) = B_1 e^{\alpha_0 t} t + C_1 e^{\alpha_0 t} + B_2 e^{-\alpha_0 t} t + C_2 e^{-\alpha_0 t}, \tag{3.41}$$

where B_1, B_2, C_1 and C_2 are constants, $\alpha_0 \neq 0$ is defined by (3.9). Using formulas (3.25) and (3.27) and substituting

$$\alpha_0 = \frac{\sqrt{4-2\xi^2}}{2\xi}, \tag{3.42}$$

we obtain

$$E(\phi_0) = -\frac{(\xi^2-2)}{3\xi^2}\left(3C_1B_2\sqrt{4-2\xi^2}\,\xi - 3C_2B_1\sqrt{4-2\xi^2}\,\xi + 8B_1B_2\left(2\xi^2-1\right)\right). \tag{3.43}$$

The pressure is as follows

$$\begin{aligned} P(\phi_0) = -\frac{(\xi^2-2)}{3\xi^2}\Bigg(& B_2 e^{-\frac{t\sqrt{4-2\xi^2}}{\xi}}\left(B_2\left(8\xi^2 - 3t\sqrt{4-2\xi^2}\,\xi - 4\right) - 3C_2\xi\sqrt{4-2\xi^2}\right) \\ &+ B_1 e^{\frac{t\sqrt{4-2\xi^2}}{\xi}}\left(3C_1\sqrt{4-2\xi^2}\,\xi + B_1\left(8\xi^2 + 3t\sqrt{4-2\xi^2}\,\xi - 4\right)\right)\Bigg). \end{aligned} \tag{3.44}$$

3.2.2 Energy density and pressure for real α

As we have seen in section 3.1.3 for some values of parameters ξ and c eq. (3.6) has real roots. We denote as p_m the values of p_α for real $\alpha = m$,

$$p_m = m^2\left(\xi^2 - 2 + 2\xi^2 m^2\right), \tag{3.45}$$

where ξ^2 is given by (3.16). For different values of c the function p_m is presented in figure 3.

If and only if $c > 1$, then there exists the interval of $0 < m^2 < m^2_{\max}$ on which $p_m < 0$. Some part of this interval is not physical because on this part $g(m^2, c) < 0$. The straightforward calculations show that

$$p_m = -\frac{\partial g(m^2, c)}{\partial m^2}. \tag{3.46}$$

Therefore, at the point

$$m^2_{\max} = -\frac{1}{2} - \frac{1}{2}W_{-1}\left(-\frac{e^{-1}}{c}\right), \tag{3.47}$$

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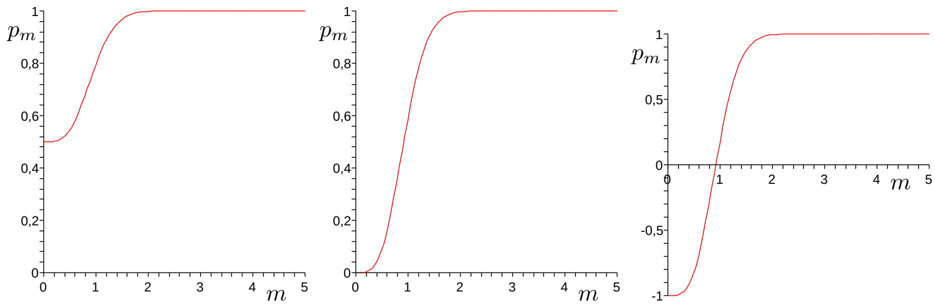

Figure 3: The dependence of p_m on m at $c = 1/2$ (right), $c = 1$ (center) and $c = 2$ (left).

ϕ	E	P
$e^{\pm mt}$	0	$p_m e^{\pm 2mt}$
$\sinh(mt)$	$p_m/2$	$p_m \cosh(2mt)/2$
$\cosh(mt)$	$-\,p_m/2$	$p_m \cosh(2mt)/2$

Table 1: Solutions, densities of energy and pressures for real α.

we obtain $p_m(m_{\max}) = 0$. We conclude that for $c > 1$ and $\xi^2 < \xi^2_{\max}$ we have two positive roots of (3.6): m_1 and $m_2 > m_1$, with $p_{m_1} < 0$ and $p_{m_2} > 0$.

The energy density and the pressure for solutions with real α one can calculate using formulas (3.38) and (3.40) and results are presented in table 1.

We see from table 1 that odd solutions are physically meaningful, $E > 0$, if p_m is positive. figure 3 shows that odd solutions are physical for $c < 1$ and any ξ^2 and for $c > 1$ only for $m^2 > m^2_{\max}$. The pressure corresponding to this solution is always positive.

Even solutions are physically meaningful if p_m is negative. Therefore, even solutions are physical only for $c > 1$ and $m^2 < m^2_{\max}$. The pressure corresponding to this solution is always negative. The equation of the state parameter for this solution is

$$w = -\cosh(2mt) < -1. \tag{3.48}$$

3.2.3 Energy density and pressure for pure imaginary α

As we have seen in section3.1.4 for some values of parameters ξ and c eq. (3.6) has only pair of pure imaginary roots. These solutions correspond to

$$\phi = C\sin(\mu(t - t_0)). \tag{3.49}$$

On the solutions (3.49) the energy and pressure are given by

$$E(C\sin(\mu(t-t_0))) = \frac{C^2}{2} p_\alpha \equiv \frac{C^2}{2}\pi_\mu, \tag{3.50}$$

$$P(C\sin(\mu(t-t_0))) = -\frac{C^2}{2}\pi_\mu \cos(2\mu(t-t_0)), \tag{3.51}$$

where

$$\pi_\mu \equiv p_{i\mu} = \mu^2\left(2 + 2\xi^2\mu^2 - \xi^2\right), \tag{3.52}$$

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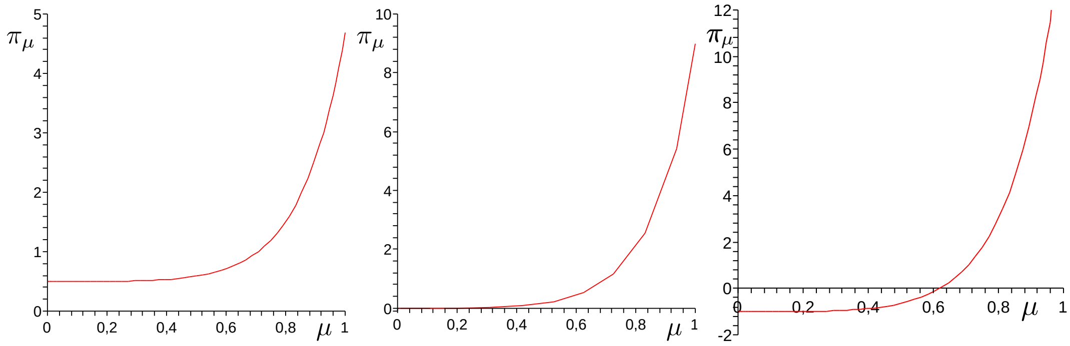


Figure 4: The dependence of π_μ on μ at $c = 1/2$ (right), $c = 1$ (center) and $c = 2$ (left).

where ξ^2 is given by (3.20). π_μ as a function of μ for different values of c is presented in Fig (4).

Note that π_μ for $c \leqslant 1$ is positive, however for $c > 1$ is positive only for $\mu^2 \geqslant \mu^2_{\text{min}} = \frac{1}{2} - \frac{1}{\xi^2}$. The equation of the state parameter for this solution is

$$w = -\cos(2\mu(t - t_0)), \quad \Rightarrow \quad |w| \leqslant 1. \tag{3.53}$$

3.2.4 Energy and pressure in the case $c = 1$

The energy density and pressure for solutions (3.14):

$$\phi_1(t) = C_1 t + C_0, \tag{3.54}$$

where C_0 and C_1 are arbitrary constants, are as follows

$$E(\phi_1) = \left(\frac{\xi^2}{2} - 1\right) C_1^2, \qquad P(\phi_1) = \left(\frac{\xi^2}{2} - 1\right) C_1^2 \tag{3.55}$$

and the state parameter $w \equiv P/E = 1$.

The straightforward calculations show that pressure and energy density for more general solutions

$$\phi = \sum_{n=1}^{N} \tilde{C}_n e^{\alpha_n t} + \phi_1 \tag{3.56}$$

are

$$E\left(\sum_{n=1}^{N} \tilde{C}_n e^{\alpha_n t} + \phi_1\right) = E\left(\sum_{n=1}^{N} \tilde{C}_n e^{\alpha_n t}\right) + E\left(\phi_1\right) \tag{3.57}$$

and

$$P\left(\sum_{n=1}^{N} \tilde{C}_n e^{\alpha_n t} + \phi_1\right) = P\left(\sum_{n=1}^{N} \tilde{C}_n e^{\alpha_n t}\right) + P\left(\phi_1\right). \tag{3.58}$$

Let, for example,

$$\phi(t) = \cosh(mt) + C_1 t + C_0.$$

The corresponding energy density and pressure are:

$$E=-\frac{1}{2}p_m+\left(\frac{\xi^2}{2}-1\right)C_1^2,\qquad P=\frac{1}{2}p_m\cosh(2mt)+\left(\frac{\xi^2}{2}-1\right)C_1^2.$$

In the case $\xi^2=2$ the root $\alpha=0$ is a double root of eq. (3.6), so eq. (3.5) has solutions (3.15):

$$\phi_2=C_3t^3+C_2t^2+C_1t+C_0.$$

We obtain:

$$E(\phi_2)=4\left(3C_1C_3-C_2^2-6C_3^2\right),\qquad P(\phi_2)=72C_3^2t^2+48C_2C_3t+4\left(C_2^2+3C_1C_3-6C_3^2\right).$$

3.2.5 Energy and pressure for complex $\alpha=r+i\nu$

For a decreasing solution

$$\phi(t)=e^{-rt}\cos(\nu t) \tag{3.59}$$

we have

$$E\left(e^{-rt}\cos(\nu t)\right)=0, \tag{3.60}$$

$$P\left(e^{-rt}\cos(\nu t)\right)=\frac{e^{-2rt}}{4}\left(p_\alpha e^{2i\nu t}+p_{\alpha^*}e^{-2i\nu t}\right). \tag{3.61}$$

Using general formulas (3.38) and (3.40) it is easy to find the energy and pressure for even and odd real solutions with complex α. For example the energy and pressure for the even solution $\phi=\cosh(rt)\cos(\nu t)$ are as follows:

$$E(\cosh(rt)\cos(\nu t))=-\frac{1}{8}\left(p_\alpha+p_{\alpha^*}\right), \tag{3.62}$$

$$P(\cosh(rt)\cos(\nu t))=\frac{p_\alpha}{16}\left(e^{-2\alpha t}+e^{2\alpha t}\right)+\frac{p_{\alpha^*}}{16}\left(e^{-2\alpha^* t}+e^{2\alpha^* t}\right). \tag{3.63}$$

The equation of state parameter on the even solution is

$$w=-\frac{p_\alpha+p_\alpha^*}{p_\alpha\cosh(\alpha t)+p_\alpha^*\cosh(\alpha^* t)}. \tag{3.64}$$

3.3 Local field representation

3.3.1 Weierstrass product and mode decomposition for the action

As in [22] we can present $F(-\Box)$ in the action (2.1) as the Ostrogradski Representation. To this purpose let us construct the Weierstrass product for the function $F(z)$ of a complex variable z. Let us recall that a complex function $R(z)$ such that its logarithmic derivative $R'(z)/R(z)$ is a meromorphic function regular in the point $z=0$, has simple poles and satisfies $|R'(z)/R(z)|<C$, $z\in\Gamma_n$, $n=1,2,\ldots$, can be presented as

$$R(z)=R(0)e^{\frac{R'(0)}{R(0)}z}\prod\left(1-\frac{z}{z_k}\right)e^{z/z_k}. \tag{3.65}$$

Γ_n, $n=1,2,\ldots$ is a set of special closed contours Γ_n such that the point $z=0$ is in all Γ_n, Γ_n is in Γ_{n+1}, and $S_n/d_n\leq C$, where S_n is a length of the contour Γ_n, and d_n is its distance from zero [51].

In the case of a more week requirement $|R'(z)/R(z)| < M|z|^p$ $z \in \Gamma_n$, $n = 1, 2, \ldots$ we have

$$R(z) = e^{f(z)} \prod \left(1 - \frac{z}{z_k}\right) e^{Q_k(z)}, \qquad Q_k(z) = \sum_{l=1}^{p} \frac{1}{l} \left(\frac{z}{z_k}\right)^l, \tag{3.66}$$

where $f(z)$ is an entire function.

In the case of $R = F$ given by (2.2) the Weierstrass product can be written in the form

$$F(\alpha^2) = e^{f(\alpha^2)} \prod_n (\alpha^2 - \alpha_n^2). \tag{3.67}$$

The function $f(z)$ in our case is

$$f(z) = A + \beta z, \tag{3.68}$$

where constants A and β are determined by ξ and c. It will be shown that the equations of motion do not depend on values of A and β.

It is convenient to pick out real roots in (3.67) and combine the complex conjugated roots:

$$F(\alpha^2) = e^{A+\beta\alpha^2} \prod (\alpha^2 - m_k^2) \prod (\alpha^2 - \alpha_n^2)(\alpha^2 - \alpha_n^{*2}), \tag{3.69}$$

where m_k denote real roots. In Subsection 3.1 we have found the cases when real roots do exist.

In the case of simple roots the Lagrangian up to a total derivative can be presented as a sum of an infinite number of fields [52]–[55]

$$\mathcal{L} = \frac{1}{2} \phi F(\partial^2) \phi \sim \frac{1}{2} \sum \left[\epsilon_n \psi_n e^{f(\partial^2)} (-\partial^2 + \alpha_n^2) \psi_n + c.c.\right], \tag{3.70}$$

where $\sim$ means equivalence up to a total derivative, ϵ_n are constants. It is the Ostrogradski representation. Note that for complex roots ψ_n are complex.

3.3.2 Mode decomposition for energy density and pressure

It is instructive to present the formula for energy and pressure obtained in section 3.2. in terms of ψ fields. All considerations below take place for nondegenerate roots.

According to a general procedure of construction of the energy and pressure we write a generalization of (3.70) to a non-flat case

$$\mathcal{L}_g = \sum \mathcal{L}_g(\psi_n), \qquad \mathcal{L}_g(\psi_n) = \frac{\epsilon_n}{2} \sqrt{-g} \psi_n e^{f(-\Box_g)} \left(\Box_g + \alpha_n^2\right) \psi_n, \tag{3.71}$$

and find

$$E_\psi = \sum_n E_n, \qquad E_n = \frac{\epsilon_n}{2} \left(\dot{\psi_n}^2 - \alpha_n^2 \psi_n^2\right) e^{f(\alpha_n^2)}, \tag{3.72}$$

$$P_\psi = \sum_n P_n, \qquad P_n = \frac{\epsilon_n}{2} \left(\dot{\psi_n}^2 + \alpha_n^2 \psi_n^2\right) e^{f(\alpha_n^2)}. \tag{3.73}$$

The E.O.M. for ψ_n is

$$(\partial^2 - \alpha_n^2) \psi_n = 0 \tag{3.74}$$

and its solutions are

$$\psi_n = A_n e^{\alpha_n t} + B_n e^{-\alpha_n t}. \tag{3.75}$$

For solutions (3.75) we obtain

$$E_\psi = 2\sum_n \alpha_n^2 \epsilon_n A_n B_n e^{\beta\alpha_n^2}, \tag{3.76}$$

$$P_\psi = \sum_n \epsilon_n \alpha_n^2 \left(A_n^2 e^{2\alpha_n t} + B_n^2 e^{-2\alpha_n t}\right) e^{\beta\alpha_n^2}. \tag{3.77}$$

On the other hand according to (3.38) and (3.40) we have

$$E = -2\sum_n A_n B_n \alpha_n^2 \left(\xi^2 - 2 + 2\xi^2\alpha_n^2\right), \tag{3.78}$$

$$P = \sum_n \left(A_n^2 e^{-2\alpha_n t} + B_n^2 e^{2\alpha_n t}\right) \alpha_n^2 \left(\xi^2 - 2 + 2\xi^2\alpha_n^2\right). \tag{3.79}$$

Comparing (3.76), (3.77) and (3.78), (3.79) and using equation (3.6) we obtain that

$$E = E_\psi, \qquad P = P_\psi \tag{3.80}$$

if and only if

$$\epsilon_n = -(2ce^{-2\alpha_n^2} + \xi^2)e^{-\beta\alpha_n^2}, \tag{3.81}$$

that is in accordance with general formula for ϵ_n [54, 22]. Note that we consider only simple roots α_n.

3.4 Finite order derivative approximation

3.4.1 Two types of approximations

There are two different types of finite order derivative approximations:

- a direct finite order derivative approximation

$$L_{2k}(\alpha^2) \equiv \sum_n^k a_n \alpha^{2n}; \tag{3.82}$$

- an approximation by a finite number of terms in the Weierstrass product

$$L_{2k}^{(W)}(\alpha^2) \equiv f(\alpha^2) \prod_n^k (\alpha^2 - \alpha_n^2). \tag{3.83}$$

We label roots so that $|\alpha_n| \leqslant |\alpha_{n+1}|$.

Locations of roots of the characteristic equation in the complex α-plane are presented in figure 5. One can see that structures of roots location for different values of ξ look similar.

Let us consider the most simple case $\xi = 0$, $c = 1$, that corresponds to

$$L(\alpha^2) \equiv L_{0,1}(\alpha^2) = 1 - e^{-2\alpha^2}. \tag{3.84}$$

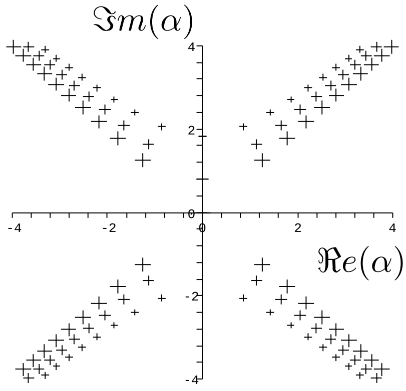


Figure 5: Roots for $c = 1$ and $\xi = 0$ (big crosses), $\xi = 2$ (middle crosses) and $\xi = 15$ (small crosses).

In this case all roots can be written explicitly

$$1 - e^{-2\alpha^2} = 2\alpha^2 e^{-\alpha^2} \prod_{j=1}^{\infty} \left(1 + \frac{\alpha^4}{\pi^2 j^2}\right). \tag{3.85}$$

Note that all conclusions can be generalized on the case of arbitrary ξ and c such that all roots α_n are simple.

3.4.2 Direct finite order approximation

In the second order derivative approximation we should keep in (3.5) with $c = 1$, $\xi = 0$ only the second derivatives

$$2\partial^2 \phi = 0. \tag{3.86}$$

This equation has the following solutions

$$\phi = C_1 t + C_2. \tag{3.87}$$

These solutions correspond to roots $\alpha_1 = 0$.

In the fourth order derivative approximation eq. (3.6) at $c = 1$ and $\xi = 0$ is as follows

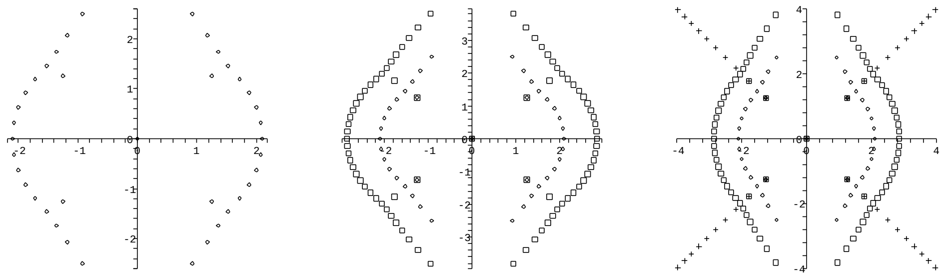
$$2\alpha^2 - 2\alpha^4 = 0 \tag{3.88}$$

and has two solutions:

$$\alpha^2 = 0, \qquad \alpha^2 = 1. \tag{3.89}$$

We see that in the fourth order direct approximation the approximate equation (3.88) has a root ($\alpha = 1$) that is absent in the initial equation (3.6).

The similar situation takes place for higher order approximations. The characteristic equation of the direct n-order approximation contains several artificial roots. The appearance of these roots is related to artificial roots of polynomial approximations of the function f in the Weierstrass product (3.67). In figure 6 we plot all roots of an n-order polynomial approximation of function $L_{0,1}(\alpha^2)$ for n=20 and 40. We see that these polynomials have true roots as well as artificial roots that go to infinity when the order of the approximate polynomial increases.



(a) (b) (c)

Figure 6: A. Roots of the 20-th order approximation $L_{0,1;10}(\alpha)$ of the function $L_{0,1}(\alpha^2)$ depicted by small diamonds. There are one root in the origin, 4 roots located on a "cross" and 15 roots on two bow-like curves. B. Roots of the 40-th order approximations, $L_{0,1;20}(\alpha)$, and roots of $L_{0,1;10}(\alpha)$ depicted by small boxes and small diamonds, respectively. $L_{0,1,20}(\alpha)$ has one root in the origin, 8 roots located on a "cross" and 4 of them coincide with true roots of $L_{0,1;10}(\alpha)$. Artificial roots of $L_{0,1,20}(\alpha^2)$ are located on bigger "bows" as compare with artificial roots of $L_{0,1,10}(\alpha^2)$. C. Exact roots of $L_{0,1}(\alpha)$ are located on a cross and depicted by small crosses. The roots of the $L_{0,1;10}(\alpha)$ and $L_{0,1;20}(\alpha)$, are depicted by small diamonds and small boxes, respectively, as in A and B.

3.4.3 Finite order weierstrass product approximation

We consider an approximation to $L_{0,1}(\alpha^2)$ that keeps only a finite number of terms in the Weierstrass product

$$L^{(W)}_{0,1,2+4k}(\alpha^2) \equiv 2\alpha^2 e^{-\alpha^2} \prod_{j=1}^{k} \left(1 + \frac{\alpha^4}{\pi^2 j^2}\right). \tag{3.90}$$

We call this approximation as (2k+1)-roots approximation. The corresponding Lagrangian is

$$\mathcal{L}^{(W)}_{0,1,2+4k} \equiv \frac{1}{2}\phi\partial^2 e^{-\partial^2} \prod_{j=1}^{k} \left(1 + \frac{\partial^4}{\pi^2 j^2}\right)\phi. \tag{3.91}$$

Let us consider one root approximation

$$\mathcal{L}^{(W)}_{0,1,2} \equiv \frac{1}{2}\phi\partial^2 e^{-\partial^2}\phi. \tag{3.92}$$

E.O.M. is

$$\partial^2 e^{-\partial^2}\phi = 0 \tag{3.93}$$

and it has an unique solution (the same as E.O.M. of 2-nd order derivative approximation)

$$\phi = At + C. \tag{3.94}$$

The next approximation is 3 roots approximation

$$\mathcal{L}^{(W)}_{0,1,6} \equiv \frac{1}{2}\phi\partial^2 e^{-\partial^2}\left(1 + \frac{\partial^4}{\pi^2}\right)\phi. \tag{3.95}$$

E.O.M. is

$$\partial^2 \left(1 + \frac{\partial^4}{\pi^2}\right) e^{-\partial^2}\phi = 0 \qquad \Leftrightarrow \qquad \partial^2 \left(1 + \frac{\partial^4}{\pi^2}\right)\phi = 0 \tag{3.96}$$

and it has the following solutions

$$\phi_6(t) = A_1 t + \delta_1 + A_2 e^{\sqrt{\frac{\pi}{2}}\,t} \sin\left(\sqrt{\frac{\pi}{2}}\,t + \delta_2\right) + A_3 e^{-\sqrt{\frac{\pi}{2}}\,t} \sin\left(\sqrt{\frac{\pi}{2}}\,t + \delta_3\right), \tag{3.97}$$

where A_k and δ_k are arbitrary constants.

The next approximation contains 4 extra roots

$$\mathcal{L}^{(W)}_{0,1,10} \equiv \frac{1}{2}\phi\partial^2 e^{-\partial^2}\left(1 + \frac{\partial^4}{\pi^2}\right)\left(1 + \frac{\partial^4}{4\pi^2}\right)\phi. \tag{3.98}$$

E.O.M. is

$$\partial^2 \left(1 + \frac{\partial^4}{\pi^2}\right)\left(1 + \frac{\partial^4}{4\pi^2}\right) e^{-\partial^2}\phi = 0 \tag{3.99}$$

and it has the following solutions

$$\phi_{10}(t) = \phi_6(t) + A_4 e^{\sqrt{\pi}\,t}\sin(\sqrt{\pi}\,t + \delta_4) + A_5 e^{-\sqrt{\pi}\,t}\sin(\sqrt{\pi}\,t + \delta_5). \tag{3.100}$$

It is obvious that solutions of the Weierstrass approximate equations reproduce a finite number of modes of the full equation.

If we restrict ourself to decreasing solutions

$$\phi_{10,\mathrm{decr}}(t) = \delta_1 + A_3 e^{-\sqrt{\frac{\pi}{2}}\,t}\sin\left(\sqrt{\frac{\pi}{2}}\,t + \delta_3\right) + A_5 e^{-\sqrt{\pi}\,t}\sin(\sqrt{\pi}\,t + \delta_5) \tag{3.101}$$

we see that the last term can be ignored as compared with term first two terms.

We can see that the Weierstrass product approximation is more preferable than direct approximation, because there is no problem with extra roots. In the next section we will use the Weierstrass product approximation to construct local cosmological models.

4. Non-flat dynamics

4.1 Modified action

The goal of this section is to consider the nonlocal model (2.1) in the Friedmann Universe. To consider the dynamics in such a system we need to solve nonlinear Friedmann equations (2.7), which represent hopelessly complicated problem. From (2.7) we obtain the following nonlinear integral equation in $H(t)$:

$$\dot{H} = -\frac{1}{2m_p^2}(\mathcal{P} + \mathcal{E}) = -\frac{1}{m_p^2}\left(\frac{\xi^2}{2}(\partial\phi)^2 - c\int_0^1 \left(\partial e^{(1+\rho)\mathcal{D}}\phi\right)\left(\partial e^{(1-\rho)\mathcal{D}}\phi\right) d\rho\right), \tag{4.1}$$

where $\mathcal{D} \equiv -\partial_t^2 - 3H(t)\partial_t$. There is no method to solve eq. (4.1), even if the function $\phi(t)$ is given.

As a first step to consider such a problem we would like to construct some effective exactly solvable models that can be consider as an approximation to the nonlocal model. To do this we use the finite order approximations, constructed by the Ostrogradski method. In other words we choose a special solution of eq. (3.5) and find the corresponding Ostrogradski approximation in the flat space-time. After we deform the obtained approximate model to the case of the Friedmann Universe, assuming that exact solutions in the Friedmann metric are coincide with exact solutions in the flat space-time. Note that the similar assumption has been used in the papers [37, 18, 38] to construct effective local models with exact solutions.

Our starting point is the Lagrangian (3.70). The corresponding action in the non-flat space-time is as follows:

$$S_{Ostr.} = \int d^4x\sqrt{-g}\left(\frac{m_p^2}{2}R + \sum_n \left[\frac{\epsilon_n}{2}\psi_n e^{\beta\alpha_n^2}(\Box_g + \alpha_n^2)\psi_n + c.c.\right] - \mathcal{V}(\psi_1, \dots, \psi_n)\right). \tag{4.2}$$

If the fields ϕ_n depend only on time and the metric is a spatially flat Friedmann metric, then we have the following equation for ψ_n

$$\epsilon_n\left(\mathcal{D} + \alpha_n^2\right)\psi_n - e^{-\beta\alpha_n^2}\mathcal{V}'_{\psi_n} = 0 \quad \Leftrightarrow \quad (2ce^{-2\alpha_n^2} + \xi^2)\left(\mathcal{D} + \alpha_n^2\right)\psi_n + \mathcal{V}'_{\psi_n} = 0, \tag{4.3}$$

where $\mathcal{V}'_{\psi_n}$ is a derivative of $\mathcal{V}$ on ψ_n. Note that form of $\mathcal{V}(\psi_1, \dots, \psi_n)$ depends on choose of special solutions ψ_1, …, ψ_n. The form of $\mathcal{V}(\psi_1, \dots, \psi_n)$ is given below (Subsection 4.4).

The energy and the pressure density in the Friedmann metric have the form

$$\mathcal{E}_{\mathrm{mod}} = E_\psi + \mathcal{V}, \tag{4.4}$$

$$\mathcal{P}_{\mathrm{mod}} = P_\psi - \mathcal{V}, \tag{4.5}$$

where E_ψ and P_ψ are given by formulas (3.72) and (3.73) respectively. This means that the extra term $\mathcal{V}$ play a role of a potential term.

The Friedmann equations of motion are

$$\begin{aligned} 3H^2 &= \frac{1}{m_p^2}\ \left(E_\psi + \mathcal{V}\right), \\ 3H^2 + 2\dot{H} &= -\frac{1}{m_p^2}\ \left(P_\psi - \mathcal{V}\right), \end{aligned} \tag{4.6}$$

Therefore

$$\dot{H} = -\frac{1}{2m_p^2}\left(P_\psi + E_\psi\right). \tag{4.7}$$

We choose such $\mathcal{V}$ that ψ_k in the non-flat case are the same as in the flat case. Using (3.76) and (3.77)) we get

$$\dot{H} = \frac{1}{2m_p^2}\sum_n \alpha_n^2\epsilon_n e^{\beta\alpha_n^2}\Big(A_n^2 e^{2\alpha_n t} + 2A_nB_n + B_n^2 e^{-2\alpha_n t}\Big). \tag{4.8}$$

Using (3.75) we can rewrite (4.8) as follows

$$\dot{H} = \frac{1}{2m_p^2}\sum_n \epsilon_n e^{\beta\alpha_n^2}\dot{\phi}_n^2 = -\frac{1}{2m_p^2}\sum_n\left(2ce^{-2\alpha_n^2} + \xi^2\right)\dot{\phi}_n^2. \tag{4.9}$$

Substituting values of ϵ_n (formula 3.81) and using formulas (3.38) and (3.40), we obtain that

$$\dot{H} = \frac{1}{2m_p^2}\left(\frac{\xi^2}{2}(\partial\phi)^2 - c\int_0^1 \left(\partial(e^{-\rho\partial^2}\Phi)\right)\left(\partial(e^{\rho\partial^2}\Phi)\right)d\rho\right) = \frac{1}{2m_p^2}(E(\phi)+P(\phi)), \quad (4.10)$$

where

$$\phi(t) = \sum_n \phi_n(t) = \sum_n \left(A_n e^{\alpha_n t} + B_n e^{-\alpha_n t}\right). \quad (4.11)$$

Therefore,

$$H(t) = \frac{1}{2m_p^2}\left(\sum_n 2A_nB_n p_{\alpha_n} t - \sum_n\left(-\frac{A_n^2}{2\alpha_n}e^{-2\alpha_n t} + \frac{B_n^2}{2\alpha_n}e^{2\alpha_n t}\right)p_{\alpha_n}\right) + H_0, \quad (4.12)$$

where H_0 is an integration constant and we assume the sum goes over the complex conjugated roots.

It is convenient to rewrite (4.9) as follows

$$\dot{H} = -\frac{1}{2m_p^2}\sum_n \frac{p_{\alpha_n}}{\alpha_n^2}\dot{\phi}_n^2 = -\frac{1}{2m_p^2}\Big((\xi^2-2)\sum_n \dot{\phi}_n^2 + 2\xi^2\sum_n \alpha_n^2\dot{\phi}_n^2\Big). \quad (4.13)$$

Thus to obtain the crossing of cosmological constant barrier one should consider the case $\xi^2 < 2$ and the field $\phi(t)$, which consists of at least two modes. It is easy to see that $H(t)$ has no singular point at finite time. For some values of parameters we obtain bouncing solutions, which satisfy the conditions $H(0) = 0$ and $\dot{H}(0) > 0$.

In the following subsections we construct effective potentials for one-, two- and N-modes solutions. One-mode models can be consider as toy-models, whereas two-fields models are more realistic. Note that there are modern inflation models, for example [56, 57], which include two scalar fields (see also [58] and references therein). In this paper we describe a procedure to construct effective models and analyse only the simplest properties of them. More detail analysis and compare with the nonlocal model dynamic is a subject of our future investigations.

4.2 One mode solutions

4.2.1 One real root: general case

Let α is a real root of (3.6), without loss of generality we can assume $\alpha > 0$. The corresponding scalar field is (see (3.75)) as follows

$$\psi(t) = Ae^{\alpha t} + Be^{-\alpha t}. \quad (4.14)$$

The function $\psi(t)$ is a solution of the following first order differential equation:

$$\dot{\psi}^2 = \alpha^2\left(\psi^2 - 4AB\right). \quad (4.15)$$

Using superpotential method [59] (see also [37, 38]) we consider the Hubble parameter as a function of ψ:

$$H = W(\psi(t)) \qquad \Rightarrow \qquad \dot{H} = \frac{dW}{d\psi}\dot{\psi}. \quad (4.16)$$

From (4.13) we obtain

$$\frac{dW}{d\psi} = -\frac{p_\alpha}{2m_p^2\alpha^2}\dot{\phi} = \mp\frac{p_\alpha}{2m_p^2\alpha}\sqrt{\psi^2 - 4AB} \tag{4.17}$$

and

$$W(\psi) = \mp\frac{p_\alpha}{2m_p^2\alpha}\left(\frac{\psi}{2}\sqrt{\psi^2 - 4AB} - 2AB\ln\left(\alpha\left(\psi + \sqrt{\psi^2 - 4AB}\right)\right)\right) + H_0. \tag{4.18}$$

The corresponding potential $V(\psi)$ is

$$\mathcal{V}(\psi) = 3m_p^2W^2 - \frac{p_\alpha}{2\alpha^2}\dot{\psi}^2 = 3m_p^2W^2 - \frac{p_\alpha}{2}\left(\psi^2 - 4AB\right). \tag{4.19}$$

We see that the potential depend on values of constants A and B, more exactly depend on value of the production AB, and does not depend on sign of $W(\psi)$. The potential $V(\psi)$ is not a polynomial, but in the limit of the flat space-time ($m_p^2 \to \infty$) we obtain a quadratic polynomial. Note that in the case of the flat space-time one can eliminate constants A and B adding a constant to potential $V(\psi)$, so in this case the equations of motion do not depend on these constants. In the following subsections we analyse in detail a few particular cases.

4.2.2 One real root: decreasing solution

Let us consider a simplest particular case:

$$\psi = \exp(-mt), \tag{4.20}$$

From (4.12) we have

$$H(t) = \frac{p_m}{4m_p^2m}e^{-2mt} + H_0. \tag{4.21}$$

Therefore to ensure that (4.20) and (4.21) solve the Friedmann equations we have to add to the action the following potential

$$\mathcal{V}(\psi) = \frac{3}{16m_p^2m^2}p_m^2\psi^4 + \frac{3p_m}{2m}H_0\psi^2 - \frac{p_m}{2}\psi^2 + 3m_p^2H_0^2. \tag{4.22}$$

Let us note that one gets the same potential for the unbounded solution $\psi_1 = \exp(mt)$. The decreasing solution (4.20), (4.21) corresponds to the scale factor

$$a(t) = a_0\exp\left(-\frac{p_m}{8m_p^2m^2}\exp(-2mt) + H_0t\right). \tag{4.23}$$

This solution has no singularities at finite t. It describes an increasing Universe only in the case $p_m < 0$. As we have seen in section 3.2.2 the negative pressure is possible if $c > 1$ and $m^2 < m_{\max}^2$ that takes place if $\xi^2 \le \xi_{\max}^2$, where $\xi_{\max}$ is given by (3.18).

4.2.3 One real root: odd solution

Let us consider a odd solution

$$\psi = \sinh(mt). \tag{4.24}$$

From (4.12) we have

$$H(t) = -\frac{p_m}{8m_p^2 m}\sinh(2mt) - \frac{p_m}{4m_p^2}t + H_0 \tag{4.25}$$

and the potential for $H_0 = 0$ is

$$\mathcal{V}(\psi) = \frac{p_m}{2}\left(-1 + \frac{3p_m}{8m_p^2}\left(t + \frac{1}{2m}\sinh(2mt)\right)^2\right). \tag{4.26}$$

The potential as a function of ψ_2 is given by

$$\mathcal{V}(\psi) = -\frac{p_m}{2} + \frac{3p_m^2}{16m_p^2 m^2}\left[\psi^2(1+\psi^2) + 2\psi\sqrt{1+\psi^2}\,\mathrm{arcsinh}(\psi) + \mathrm{arcsinh}(\psi)^2\right]. \tag{4.27}$$

The odd increasing solution (4.24), (4.25) corresponds to the scale factor

$$a(t) = a_0 \exp\left(-\frac{p_m}{16m_p^2 m^2}\left(\cosh(2mt) + 2m^2t^2\right) + H_0 t\right). \tag{4.28}$$

The scale factor (4.28) has no singularity. It increases at large time if $p_m < 0$.

4.2.4 One pure imaginary root

Let us consider an odd solution

$$\psi = \sin(\mu t). \tag{4.29}$$

From (3.50), (3.51) and (4.7) we get explicitly the Hubble parameter

$$H(t) = \frac{\pi_\mu}{4m_p^2}t + \frac{\pi_\mu}{8\mu m_p^2}\sin(2\mu t) + H_0. \tag{4.30}$$

The potential for $H_0 = 0$ as a function of ψ is given by

$$\mathcal{V}(\psi) = \frac{1}{2}\pi_\mu + \frac{3\pi_\mu^2}{16m_p^2\mu^2}\left[\psi^2(1-\psi^2) + 2\psi\sqrt{1-\psi^2}\,\arcsin(\psi) + \arcsin(\psi)^2\right]. \tag{4.31}$$

Note, that this formula is valued only for $\psi^2 \leqslant 1$. On this region the potential (4.31) is convex and it has an unique minimum.

The odd periodic solution (4.29), (4.30) corresponds to the scale factor

$$a(t) = a_0 \exp\left(-\frac{\pi_\mu}{16\mu^2 m_p^2}\cos(2\mu t) + \frac{\pi_\mu}{8m_p^2}t^2 + H_0 t\right) \tag{4.32}$$

and we have an expansion with an acceleration if $\pi_\mu > 0$. In this case at $H_0 = 0$ we obtain the bouncing solution.

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4.3 Solutions with the crossing of cosmological constant barrier

4.3.1 Pair of complex roots

For the case of a complex root $\alpha = r + i\nu$ we consider the following real solution

$$\psi_4 = \psi_\alpha + \psi_{\alpha^*}, \qquad \text{where} \qquad \psi_\alpha = e^{-\alpha t}, \qquad \psi_{\alpha^*} = e^{-\alpha^* t}. \tag{4.33}$$

From (4.12) we have

$$H(t) = \frac{1}{4m_p^2}\left[\frac{p_\alpha}{\alpha}e^{-2\alpha t} + \frac{p_{\alpha^*}}{\alpha^*}e^{-2\alpha^* t} + H_0\right] = \frac{1}{4m_p^2}\left[\frac{p_\alpha}{\alpha}\psi_\alpha^2 + \frac{p_{\alpha^*}}{\alpha^*}(\psi_{\alpha^*}^*)^2 + H_0\right]. \tag{4.34}$$

The corresponding potential is as follows

$$\begin{aligned}\mathcal{V} = \frac{3}{16m_p^2}\Big[&\alpha^2\left(\xi^2 - 2 + 2\xi^2\alpha^2\right)^2\psi_\alpha^4 + \alpha^{*2}\left(\xi^2 - 2 + 2\xi^2\alpha^{*2}\right)\psi_{\alpha^*}^4 + H_0^2 + \\ &+2\alpha\alpha^*\left(\xi^2 - 2 + 2\xi^2\alpha^2\right)\left(\xi^2 - 2 + 2\xi^2\alpha^{*2}\right)\psi_\alpha^2\psi_{\alpha^*}^2 + 2H_0\left(\frac{p_\alpha}{\alpha}\psi_\alpha^2 + \frac{p_{\alpha^*}}{\alpha^*}(\psi_{\alpha^*}^*)^2\right)\Big].\end{aligned}$$

From (4.34) it follows that

$$\begin{aligned}\dot{H} &= -\frac{1}{2m_p^2}\left[p_\alpha e^{-2\alpha t} + p_{\alpha^*}e^{-2\alpha^* t}\right] \\ &= -\frac{e^{-2rt}}{2m_p^2}\Big[(p_\alpha + p_{\alpha^*})\cos(\nu t) + i(p_\alpha - p_{\alpha^*})\sin(\nu t)\Big].\end{aligned} \tag{4.35}$$

It is easy to check that $p_{\alpha^*} = p_\alpha^*$, so $q_1 \equiv p_\alpha + p_{\alpha^*}$ and $q_2 \equiv i(p_\alpha - p_{\alpha^*})$ are real numbers. Formula (4.35) can be rewrite in the following form:

$$\dot{H} = -\frac{\sqrt{q_1^2 + q_2^2}}{2m_p^2}e^{-2rt}\sin(\nu t + \gamma), \tag{4.36}$$

where γ is defined by the following relations:

$$\sin(\gamma) = \frac{q_1}{\sqrt{q_1^2 + q_2^2}}, \qquad \cos(\gamma) = \frac{q_2}{\sqrt{q_1^2 + q_2^2}}. \tag{4.37}$$

As known the state parameter

$$w = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}, \tag{4.38}$$

so we obtain that w crossing the barrier $w = -1$ infinite number of times.

For example, in the case: $\xi^2 = 0$ and $c = 1$, when $p_\alpha = -2\alpha^2$ and

$$\dot{H} = \frac{1}{m_p^2}\sum_n \dot{\phi}_n^2. \tag{4.39}$$

If we choose $\alpha = \sqrt{\pi} + i\sqrt{\pi}$, then $p_\alpha = -2\pi i$, $q_1 = 0$, $q_2 = -4\pi$, $\gamma = \pi$ and

$$\dot{H} = \frac{1}{m_p^2}\left(\alpha^2 e^{-2\alpha t} + (\alpha^*)^2 e^{-2\alpha^* t}\right) = -\frac{4\pi}{m_p^2}e^{-\sqrt{\pi}t}\sin(\sqrt{\pi}t - \pi). \tag{4.40}$$

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The Hubble parameter (4.34) corresponds to the following scale factor

$$a(t) = a_0 \exp\left\{H_0 t - \frac{1}{8m_p^2}\left[\frac{p_\alpha}{\alpha^2}e^{-2\alpha t} + \frac{p_{\alpha^*}}{\alpha^{*2}}e^{-2\alpha^* t}\right]\right\}, \qquad (4.41)$$

substituting the explicit formula for p_α and p_{α^*} we get

$$a(t) = a_0 \exp\left\{H_0 t - \left(\frac{(\xi^2-2)}{4m_p^2}\cos(2\nu t) - \frac{\xi^2 e^{-2rt}}{2m_p^2}\left[(r^2-\nu^2)\cos(2\nu t) - 2r\nu\sin(2\nu t)\right]\right)\right\}. \qquad (4.42)$$

We see that a late time expansion regime corresponds only to $H_0 > 0$ (compare with [20]). Let us note that a constant part of the Hubble parameter H_0 can be incorporated in a plane solutions [20]:

$$\psi_n(t) = A_n e^{\alpha_{n,H_0} t} + B_n e^{-\alpha_{n,H_0} t}, \qquad (4.43)$$

where α_{n,H_0} is related with α_n as

$$\alpha_{n,H_0}^2 + 3H_0\alpha_{n,H_0} = \alpha_n^2. \qquad (4.44)$$

Using this approximations authors has been investigate the case of the SFT inspired values of parameters ξ^2 and c and obtain a cosmic acceleration with a periodic crossing of the $w = -1$ barrier. In our paper we obtain the similar result for arbitrary values of ξ^2 and c. Note that for some particular values of these parameters we can obtain one-fold crossing of the $w = -1$ barrier.

4.3.2 Two real roots solutions

The above-mentioned solutions for real roots m_n correspond to monotonic behaviour of the Hubble parameter. To describe nonmonotonic behaviour let us consider the case of $c > 1$ and $\xi^2 < \xi^2_{\max}$. There exist two real roots of (3.6) m_1 and m_2 such that that $|m_1| < |m_2|$. The corresponding solution to (3.5) is

$$\psi = \psi_1 + \psi_2, \quad \text{where} \quad \psi_1 = Ae^{m_1 t}, \quad \psi_2 = Be^{m_2 t}, \qquad (4.45)$$

where A and B are constants. Without loss of generality we can put $A = 1$.

Using (4.8) we obtain

$$\dot{H} = -\frac{1}{2m_p^2}\left(p_{m_1}e^{2m_1 t} + B^2 p_{m_2}e^{2m_2 t}\right). \qquad (4.46)$$

Let us analyze a possibility $\dot{H} = 0$, which correspond to crossing of the cosmological constant barrier for the state parameter w. In Subsection 3.2.2 we have obtained that $p_{m_1} < 0$ and $p_{m_2} > 0$ (See figures 1 and 3). So, for any roots m_1 and m_2, there exist such real B that $\dot{H} = 0$ at the point $t = t_1$:

$$B = \pm\sqrt{\frac{-\,p_{m_1}e^{2m_1 t_1}}{p_{m_2}e^{2m_2 t_1}}}. \qquad (4.47)$$

We conclude that solutions (4.45) correspond to cosmological models with the crossing of $w = \,-1$ barrier.

The Hubble parameter and the scale factor are as follows:

$$\begin{aligned}
H \,=\, & -\frac{1}{4m_p^2 m_1 m_2}\left(p_{m_1} m_2 e^{2m_1 t} + B^2 p_{m_2} m_1 e^{2m_2 t}\right) + H_0, \\
a \,=\, & a_0 \exp\left(-\frac{1}{4m_p^2 m_1^2 m_2^2}\left(p_{m_1} m_2^2 e^{2m_1 t} + B^2 p_{m_2} m_1^2 e^{2m_2 t}\right) + H_0 t\right).
\end{aligned} \tag{4.48}$$

If $m_1 > 0$ and $m_2 < 0$, then at late time

$$\dot{H} \approx \,-\frac{1}{2m_p^2} p_{m_1} e^{2m_1 t} > 0 \tag{4.49}$$

and

$$H \approx \,-\frac{1}{4m_p^2 m_1} p_{m_1} e^{2m_1 t} > 0. \tag{4.50}$$

Using

$$H = W(\psi_1, \psi_2) = \frac{-1}{4m_p^2 m_1 m_2}\left(p_{m_1} m_2 \psi_1^2 + p_{m_2} m_1 \psi_2^2\right) + H_0, \tag{4.51}$$

we obtain the fourth degree polynomial potential

$$\mathcal{V}(\psi_1, \psi_2) = 3m_p^2\left(\frac{1}{4m_p^2 m_1 m_2}\left(p_{m_1} m_2 \psi_1^2 + p_{m_2} m_1 \psi_2^2\right) - H_0\right)^2 - \frac{1}{2}\left(p_{m_1}\psi_1^2 + p_{m_2}\psi_2^2\right). \tag{4.52}$$

We can conclude that all solutions (4.45) correspond to cosmological models with the crossing of $w = \,-1$ barrier. Let us remind in this context that models with a crossing of the $w = -1$ barrier are a subject of recent studies [40, 41, 60 – 63]. Simplest models include two scalar fields (one phantom and one usual field, see [38, 64, 65] and refs. therein). In our case a nonlocality provides a crossing of the $w = -1$ barrier in spite of the presence of only one scalar field. This fact has a simple explanation. The crossing of the $w = -1$ in our case is driven by an equivalence of our nonlocal model to a set of local models some of which are ghosts.

4.4 Cosmological models for N-mode solutions

Let us construct a cosmological model, with $\dot{H}$, defined by (4.12). To do this we use the superpotential method [59]. We consider the Hubble parameter as a function (superpotential) of ψ_n: $H(t) = W(\psi_1, \psi_2, \ldots, \psi_N)$. To construct potential we use the simplest form of the superpotential

$$H(t) = W(\psi_1, \psi_2, \ldots, \psi_N) = \sum_{n=1}^{N} W_n(\psi_n). \tag{4.53}$$

Using formula

$$\dot{H} = \sum_{n=1}^{N} \frac{dW_n}{d\psi_n}\dot{\psi}_n, \tag{4.54}$$

we obtain, that (4.9) is satisfied if

$$\frac{dW_n}{d\psi_n} = -\frac{1}{2m_p^2}\sum_n \left(2ce^{-2\alpha_n^2} + \xi^2\right)\dot{\psi}_n. \tag{4.55}$$

In the case of simple roots α_n the solutions

$$\psi_n = A_n e^{\alpha_n t} + B_n e^{-\alpha_n t} \tag{4.56}$$

satisfy the following first order differential equation:

$$\dot{\psi}_n^2 = \alpha_n^2\left(\psi_n^2 - 4A_nB_n\right). \tag{4.57}$$

So,

$$\frac{dW_n}{d\psi_n} = -\frac{p_{\alpha_n}}{2m_p^2\alpha_n^2}\dot{\psi}_n = \mp\frac{p_{\alpha_n}}{2m_p^2\alpha_n}\sqrt{\psi_n^2 - 4A_nB_n} \tag{4.58}$$

and

$$W_n(\psi_n) = \mp\frac{p_{\alpha_n}}{2m_p^2\alpha_n}\left(\frac{\psi_n}{2}\sqrt{\psi_n^2 - 4A_nB_n} - 2A_nB_n\ln\left(\alpha(\psi_n + \sqrt{\psi_n^2 - 4A_nB_n})\right)\right). \tag{4.59}$$

The corresponding potential $\mathcal{V}(\psi_1,\ldots,\psi_n)$ is

$$\mathcal{V}(\psi_1,\ldots,\psi_n) = 3m_p^2W^2 - \sum_n\frac{p_{\alpha_n}}{2\alpha_n^2}\dot{\psi}_n^2 = 3m_p^2\left(\sum_n W_n\right)^2 + \sum_n\frac{p_{\alpha_n}}{2}\left(\psi_n^2 - 4A_nB_n\right). \tag{4.60}$$

We see that the potential and does not depend on sign of W. The potential $V(\psi)$ is not a polynomial, but in the limit of the flat space-time ($m_p^2 \to \infty$) we obtain a quadratic polynomial. Formula (4.60) is a straightforward generalization of (4.19). Note that the form of potential is not unique (compare with [38]).

5. Conclusions

We have studied linear nonlocal models which violate the NEC. The form of them is inspired by the SFT. These models have an infinite number of higher derivative terms and are characterized by two positive parameters, ξ^2 and c.

The model with $c = 1$ is a toy nonlocal model for the dilaton coupling to the gravitation field. A distinguished feature of it is the invariance under the shift of the dilaton field to a constant.

For particular cases of the parameters ξ^2 and c the corresponding actions describe linear approximations to the bosonic [44, 32] and nonBPS fermionic [46] cubic SFT as well as to the nonpolynomial SFT [47, 48].

The case $\xi = 0$ corresponds to a linear approximation to the p-adic string [16]. Let us note that recently a p-adics string inflation model has been considered [23].

In the flat case all solutions of the equation of motion are plane waves and are controlled by roots of the characteristic equation. Our characteristic equation has complex distinctive simple roots. In some particular cases there are single or double roots, which are real or

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pure imaginary. The energy on plane waves is equal to zero except for the cases of couples of roots $(\alpha, -\alpha)$. The pressure is a sum of one mode pressures. The pressure for the one plane wave corresponding to a real root can be positive or negative depending on parameters of the theory. For $c \leq 1$ the one mode pressure is positive and for $c > 1$ it could be negative or positive.

To study cosmological applications we have investigated the behaviour of the models in the Friedmann background. We have performed this study within an approximation scheme. A simplest approximation is a local field approximation (or a mechanical analogous model in a terminology of [18]). On an example of the free flat case we have shown that in special cases we can use a local two derivatives approximation, but the next derivative approximations exhibit artifacts. Followed [22] we have used the Weierstrass product representation to study finite mode approximations. As was noted in [22] a straightforward application of the Ostrogradski method to these approximations indicates that energies are unbounded (an eigenvalue problem for the unbounded hyperbolic Klein-Gordon equation on manifolds is solved in [66]) and it is expected [22] that an incorporation of non-flat metric or nonlinear terms could drastically change the situation.

A distinguished cosmological property of these models is a crossing of the phantom divide [20, 24]. But there are also possibilities for other types of behaviour. Namely, the toy dilaton model possesses decreasing solutions describing asymptotical flat Universes (adding a cosmological constant modifies these solutions to near-de Sitter solutions). It also has odd bouncing solutions describing a contracting Universes meanwhile even bouncing solutions are forbidden. For special values of parameters corresponding to tachyon SFT models there are even bouncing solutions with an accelerated expansion.

We have shown that for some particular cases there are deformations of the model such that exact solutions of the linear problem are inherited by nonlinear non-flat ones. This is similar to what was done before for local models [37, 67, 68]. A stability of exact solutions in local models has been studied in [37]. We will study stability of our solutions in the future work.

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A. Calculations of nonlocal energy density and pressure on plane waves

In this appendix we calculate the energy density and pressure for the following solution

$$\phi = Ae^{\alpha_1 t} + Be^{\alpha_2 t}, \tag{A.1}$$

where α_1 and α_2 are different roots of (3.6). We have

$$E(Ae^{\alpha_1 t}+Be^{\alpha_2 t}) = E(Ae^{\alpha_1 t})+E(Be^{\alpha_2 t})+E_{\text{cross}}(Ae^{\alpha_1 t},Be^{\alpha_2 t}) = ABE_{\text{cross}}(e^{\alpha_1 t},e^{\alpha_2 t}), \tag{A.2}$$

where the functional E_{cross} is defined as follows:

$$\begin{aligned} E_{\text{cross}}(\phi_1,\phi_2) &= \xi^2\partial\phi_1\partial\phi_2+\phi_1\phi_2+c\Phi_1\Phi_2+\\ &\quad +c\int_0^1\left[\left(e^{-\rho\partial^2}\Phi_1\right)\partial^2\left(e^{-\rho\partial^2}\Phi_2\right)+\left(e^{-\rho\partial^2}\Phi_2\right)\partial^2\left(e^{-\rho\partial^2}\Phi_1\right)\right]d\rho\\ &\quad -c\int_0^1\left[\partial\left(e^{-\rho\partial^2}\Phi_1\right)\partial\left(e^{-\rho\partial^2}\Phi_2\right)+\partial\left(e^{-\rho\partial^2}\Phi_2\right)\partial\left(e^{-\rho\partial^2}\Phi_1\right)\right]d\rho,\\ \Phi_1 &\equiv e^{-\partial^2}\phi_1,\\ \Phi_2 &\equiv e^{-\partial^2}\phi_2. \end{aligned}$$

For $\phi_1=e^{\alpha_1 t}$ and $\phi_2=e^{\alpha_2 t}$ we have

$$\begin{aligned} E_{\text{cross}}(e^{\alpha_1 t},e^{\alpha_2 t}) &= e^{(\alpha_1+\alpha_2)t}\Big\{\alpha_1\alpha_2-1+ce^{(-\alpha_1^2-\alpha_2^2)}\\ &\qquad +ce^{-\alpha_1^2-\alpha_2^2}\int_0^1\Big[\alpha_2^2e^{\rho(\alpha_2^2-\alpha_1^2)}+\alpha_1^2e^{\rho(\alpha_1^2-\alpha_2^2)}\\ &\qquad\qquad -\alpha_1\alpha_2\left(e^{\rho(\alpha_2^2-\alpha_1^2)}+e^{\rho(\alpha_1^2-\alpha_2^2)}\right)\Big]d\rho\Big\}. \end{aligned} \tag{A.3}$$

If $\alpha_1=-\alpha_2$, then it is easy to show that

$$E_{\text{cross}}(e^{\alpha_1 t},e^{-\alpha_1 t}) = -2p_{\alpha_1}. \tag{A.4}$$

where p_α is given by (3.33). We get

$$E(Ae^{\alpha_1 t}+Be^{-\alpha_1 t}) = -2ABp_{\alpha_1}. \tag{A.5}$$

In the opposite case ($\alpha_1\neq-\alpha_2$)

$$\begin{aligned} E_{\text{cross}}(e^{\alpha_1 t},e^{\alpha_2 t}) &= E_{\alpha_1,\alpha_2}e^{(\alpha_1+\alpha_2)t};\\ E_{\alpha_1,\alpha_2} &= \alpha_1\alpha_2-1\\ &\quad +\frac{ce^{(-\alpha_1^2-\alpha_2^2)}\left(\alpha_2^2e^{\alpha_2^2-\alpha_1^2}-\alpha_1^2e^{\alpha_1^2-\alpha_2^2}-\alpha_1\alpha_2\left(e^{\alpha_2^2-\alpha_1^2}-e^{\alpha_1^2-\alpha_2^2}\right)\right)}{\alpha_2^2-\alpha_1^2}. \end{aligned}$$

Constants α_1 and α_2 are roots of (3.6), therefore,

$$E_{\alpha_1,\alpha_2} = \frac{1}{\alpha_2^2-\alpha_1^2}\Big\{\alpha_1\alpha_2\left(\left(\xi^2\alpha_2^2-ce^{-2\alpha_2}\right)-\left(\xi^2\alpha_1^2-ce^{-2\alpha_1}\right)\right)+\alpha_2^2\xi^2\alpha_1^2-\alpha_1^2\xi^2\alpha_2^2\Big\}=0. \tag{A.6}$$

Note that, the equality $E_{\text{cross}}(e^{\alpha_1 t},e^{\alpha_2 t})=0$ at $\alpha_1\neq-\alpha_2$ also follows from the energy conservation low. Let us calculate the pressure $P(\phi)$ for the solution $\phi=Ae^{\alpha_1 t}+Be^{\alpha_2 t}$.

$$P\left(Ae^{\alpha_1 t}+Be^{\alpha_2 t}\right)=P\left(Ae^{\alpha_1 t}\right)+P\left(Be^{\alpha_2 t}\right)+P_{\text{cross}}\left(Ae^{\alpha_1 t},Be^{\alpha_2 t}\right), \tag{A.7}$$

where

$$
\begin{aligned}
P_{\text{cross}}(\phi_1, \phi_2) &= E_{k_{\text{cross}}}(\phi_1, \phi_2) + E_{nl2_{\text{cross}}}(\phi_1, \phi_2) - E_{p_{\text{cross}}}(\phi_1, \phi_2) - E_{nl1_{\text{cross}}}(\phi_1, \phi_2), \\
E_{k_{\text{cross}}} &= \xi^2 \partial\phi_1 \partial\phi_2, \\
E_{p_{\text{cross}}} &= \phi_1\phi_2 + c\Phi_1\Phi_2, \\
E_{nl1_{\text{cross}}} &= c\int_0^1 \left[\left(e^{-\rho\partial^2}\Phi_1\right)\partial^2\left(e^{-\rho\partial^2}\Phi_2\right) + \left(e^{-\rho\partial^2}\Phi_2\right)\partial^2\left(e^{-\rho\partial^2}\Phi_1\right)\right] d\rho,
\end{aligned}
\tag{A.8}
$$

and

$$
E_{nl2_{\text{cross}}} = -c\int_0^1 \left[\partial\left(e^{-\rho\partial^2}\Phi_1\right)\partial\left(e^{-\rho\partial^2}\Phi_2\right) + \partial\left(e^{-\rho\partial^2}\Phi_2\right)\partial\left(e^{-\rho\partial^2}\Phi_1\right)\right] d\rho. \tag{A.9}
$$

The functional $E_{\text{cross}} = E_{k_{\text{cross}}} + E_{nl2_{\text{cross}}} + E_{p_{\text{cross}}} + E_{nl1_{\text{cross}}}$, proving that $E_{\text{cross}} = 0$, we obtain that

$$
E_{k_{\text{cross}}} + E_{nl2_{\text{cross}}} = 0 \qquad \text{and} \qquad E_{p_{\text{cross}}} + E_{nl1_{\text{cross}}} = 0. \tag{A.10}
$$

Therefore

$$
P_{\text{cross}}\left(e^{\alpha_n t}, e^{\alpha_k t}\right) = 0, \quad \text{if} \quad \alpha_n \neq -\alpha_k. \tag{A.11}
$$

The straightforward calculations give that

$$
P_{\text{cross}}(e^{\alpha_n t}, e^{-\alpha_n t}) = 0. \tag{A.12}
$$

So, $P_{\text{cross}}(e^{\alpha_n t}, e^{\alpha_k t}) = 0$ for all α_n and $\alpha_k \neq \alpha_n$. The pressure is as follows:

$$
P\left(\sum_{n=1}^{2} C_n e^{\alpha_n t}\right) = \sum_{n=1}^{2} C_n^2 P\left(e^{\alpha_n t}\right), \tag{A.13}
$$

where

$$
P\left(e^{\alpha t}\right) = p_\alpha e^{2\alpha t}. \tag{A.14}
$$

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